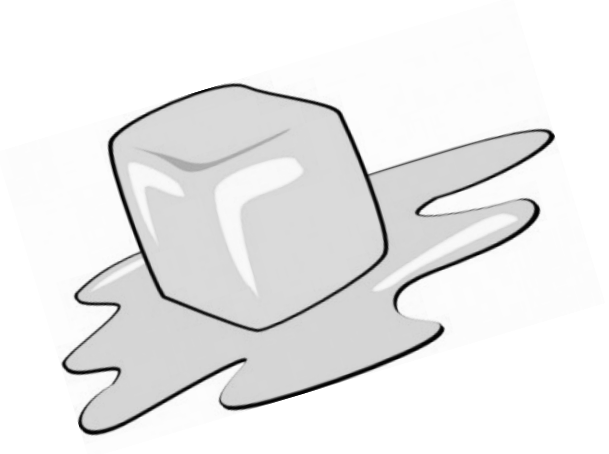
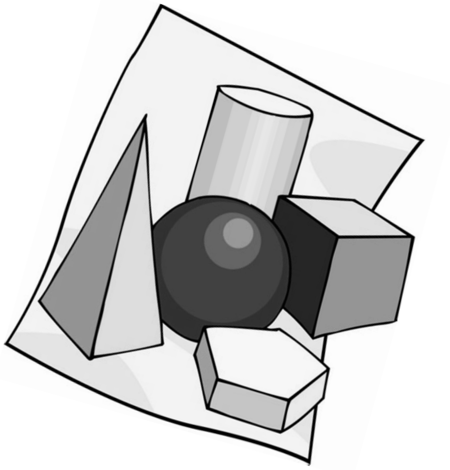
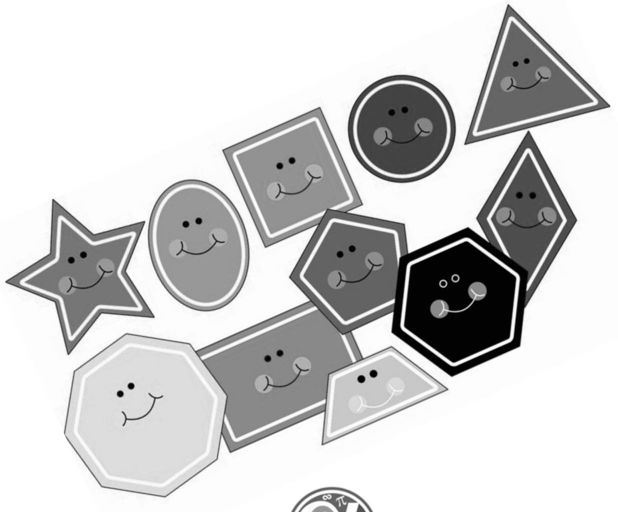
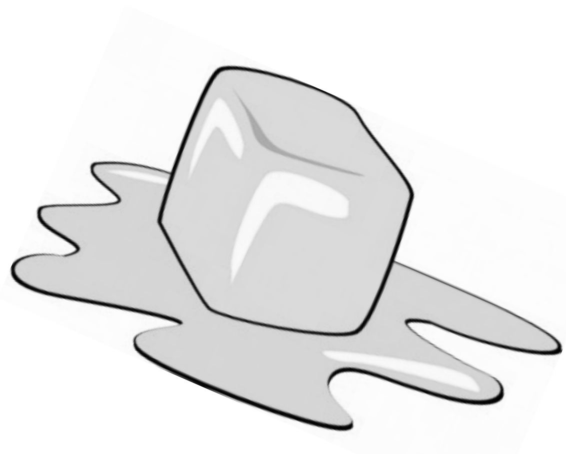



GEOMETRY (COMMON CORE)

FACTS YOU MUST KNOW COLD FOR THE REGENTS EXAM

Notes to the Student

What is this? How do I use it to study?

Welcome to the “Geometry (Common Core) Facts You Must Know Cold for the Regents Exam” study guide! I hope that you find this guide to be an invaluable resource as you are studying for your Geometry Regents examination. This guide holds the essential information, formulas, and concepts that you *must know* in order to pass, or even master, your Regents exam! Over 200 hours have been put into the development of this study guide – from the clipart, to formatting, and from the colors to the mathematical theorems and concepts themselves, this packet has it all for you, the student and/or teacher! This study guide is specifically designed for students but can be used by teachers to ensure that there are no gaps in their curriculum. So, students, how do you use this to be incredibly successful? First and foremost, you need to *know this stuff cold*. There are no exceptions – you need to memorize and understand the material presented in this study guide. If you don’t know the basics, then how are you going to complete practice exams? You can’t. You need to take one step at a time; this is the first step. After you have read through these concepts and theorems several times, it’s time to try an administered Geometry Regents exam. For your first attempt, I recommend that you have this study guide handy as a reference guide. If you’re stuck on a question, consult this guide to see what concept or theorem you need to apply to the problem. This method of getting stuck on a question, consulting this study guide, and finding the correct theorem helps your mind grow and retain these mathematical concepts. If you are still stuck, then visit www.nysmathregentsprep.com and watch our ***fully explained*** regents exam videos in Geometry. We have all exams available! I wish you all of mathematical success! If you have any questions, feel free to contact me at tclark@nysmathregentsprep.com. Good luck!

Notes to the Teacher

What’s new to this edition?

This is the fourth edition of the “Geometry (Common Core) Facts You Must Know Cold for the Regents Exam”, published in the spring of 2018 as a **black & white friendly version**. If you are familiar with the previous versions, you may notice some minor changes. It was discovered that a few topics were missing from the previous edition. A listing shown below indicates the missing topics from the third edition that have been added in the fourth edition:

- ✓ Speed and average speed formulas from Algebra 1
- ✓ Coordinate geometry proof properties for quadrilaterals

In addition to these topics, formatting was updated, diagrams were improved, and all typos that we were informed about were corrected. We hope that you find this study guide to be an invaluable resource for you and your students. We encourage you to make photo copies and distribute this to all of your students. If you teach other regents level courses such as Algebra 1 and Algebra 2 (and eventually AP Calculus), visit our website at www.nysmathregentsprep.com to download those study guides too! If you have any questions, comments, or suggestions, please don’t hesitate to contact me at tclark@nysmathregentsprep.com.

Like what we do?

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Dedication

I would like to dedicate this study guide to the following mathematics teachers of Farmingdale High School, who have inspired me every step of the way to fulfil my goal of becoming a mathematics teacher: Mrs. Mary–Elena D’Ambrosio, Mrs. Laura Angelo–Provenza, Mrs. Louise Corcoran, Mrs. Efstratia Vouvoudakis, Mr. Scott Drucker, and Mr. Ed Papo. Other teachers who have also inspired me include Mrs. Jacquelyn Passante–Merlo and Mrs. Mary Ann DeRosa of W. E. Howitt Middle School, and Ms. Elizabeth Bove of Massapequa High School.

I would especially like to thank Mrs. Mary–Elena D’Ambrosio and Mrs. Laura Angelo–Provenza for their suggestions and advice as to how to improve this fourth edition of the “Geometry (Common Core) Facts You Must Know Cold for the Regents Exam”. Their input was invaluable.

ANGLE, SEGMENT, & TRIANGLE RELATIONSHIPS & COORDINATE GEOMETRY

Polygons – Interior/Exterior Angles

Sum of Interior Angles: $180\,(n-2)$

Each Interior Angle of a Regular Polygon:

$$\frac{180\,(n-2)}{n}$$

Sum of Exterior Angles: $360°$

Each Exterior Angle: $\frac{360}{n}$

Triangles

Classifying Triangles

Sides:

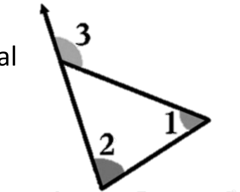
Scalene: No congruent sides
Isosceles: 2 congruent sides
Equilateral: 3 congruent sides

Angles:

Acute: All angles are < 90°
Right: One right angle that is 90°
Obtuse: One angle that is > 90°
Equiangular: 3 congruent angles (60°)

All triangles have 180°

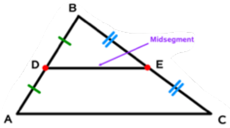
Exterior Angle Theorem: The exterior angle is equal to the sum of the two non-adjacent interior angles.



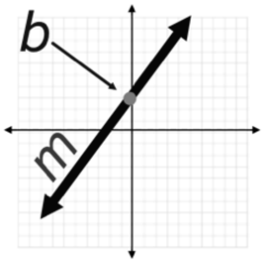
$m\angle 1 + m\angle 2 = m\angle 3$

Midsegment: a segment that joins two midpoints

- Always parallel to the third side
- $\frac{1}{2}$ the length of the third side
- Splits the triangle into two similar triangles



Coordinate Geometry

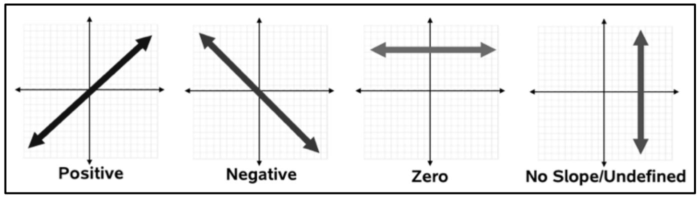


Slope-Intercept Form of a Line: $y = mx + b$
where m is the slope and b is the y-intercept.

Point-Slope Form of a Line: $y - y_1 = m(x - x_1)$
where m is the slope, and x_1 and y_1 are the values of a given point on the line.

Slope Formula: $\boldsymbol{m} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Slopes:



- **Parallel** lines have the ***same*** slope
- **Perpendicular** lines have ***negative reciprocal*** slopes (flip the fraction & change the sign)
- **Collinear points** are points that lie of the ***same*** line.

Midpoint Formula: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Segment Ratios to Partition Line Segments:

$$\frac{x - x_1}{x_2 - x} = \textit{Given Ratio} \qquad \frac{y - y_1}{y_2 - y} = \textit{Given Ratio}$$

Triangle Inequality Theorems

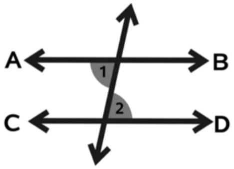
- The sum of 2 sides must be greater than the third side
- The difference of 2 sides must be less than the third side
- The longest side of the triangle is opposite the largest angle
- The shortest side of the triangle is opposite the smallest angle

Isosceles Triangle

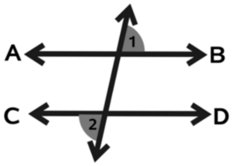
- 2 ≅ sides and 2 ≅ base angles
- The altitude drawn from the vertex is also the median and angle bisector
- If two sides of a triangle are ≅, then the angles opposite those ≅ sides are ≅

Parallel Lines

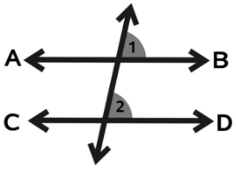
Alternate interior angles are **congruent**



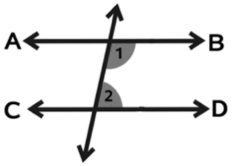
Alternate exterior angles are **congruent**



Corresponding angles are **congruent**

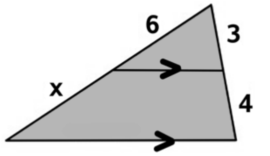


Same-side interior angles are **supplementary**



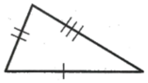
Side – Splitter Theorem

If a line is parallel to a side of a triangle and intersects the other two sides, then this line divides those two sides proportionally.

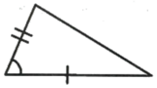


Triangle Congruence Theorems

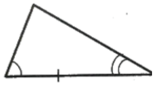
Side-Side-Side **(SSS)**



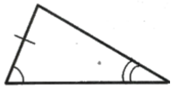
Side-Angle-Side **(SAS)**



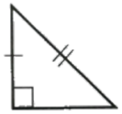
Angle-Side-Angle **(ASA)**



Angle-Angle-Side **(AAS)**



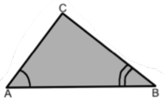
Hypotenuse-Leg **(HL)**



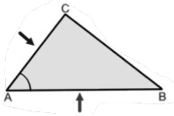
CPCTC – Corresponding Parts of Congruent Triangles are Congruent

Similar Triangle Theorems

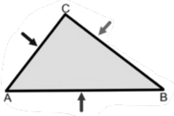
Angle-Angle **(aa)**



Side-Angle-Side **(SAS)**

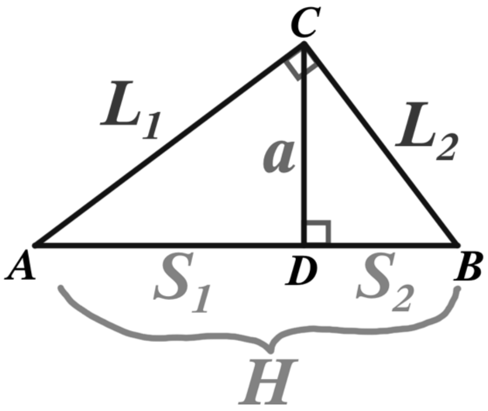


Side-Side-Side **(SSS)**



- Similar figures have congruent angles and proportional sides
- **CSSTP -** Corresponding Sides of Similar Triangles are in Proportion
- In a proportion, the product of the means equals the product of the extremes

The Mean Proportional



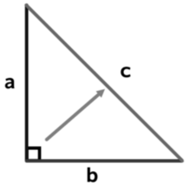
Altitude Theorem (SAAS / Heartbeat Method):
The altitude is the geometric mean between the 2 segments of the hypotenuse.

$$\frac{S_1}{a} = \frac{a}{S_2}$$

Leg Theorem (HYLLS / PSSW):
The leg is the geometric mean between the segment it touches and the whole hypotenuse.

$$\frac{S_1}{L_1} = \frac{L_1}{H} \quad \text{and} \quad \frac{S_2}{L_2} = \frac{L_2}{H}$$

The Pythagorean Theorem

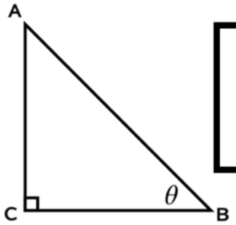


To find the missing side of any **right** triangle if two sides are given, use:

$$a^2 + b^2 = c^2$$

where *a* and *b* are the legs, and *c* is the hypotenuse

Trigonometry (SOHCAHTOA)



$\sin\theta = \frac{opposite}{hypotenuse}$	$\cos\theta = \frac{adjacent}{hypotenuse}$	$\tan\theta = \frac{opposite}{adjacent}$

- When solving for a side, use the sin, cos, and tan buttons
- When solving for an angle, use the $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ buttons
- *Recall from Algebra 1*: $\boldsymbol{Average\ Speed = \frac{\Delta Distance}{\Delta Time}}$ and $\boldsymbol{Speed = \frac{Distance}{Time}}$

Cofunctions:

- Sine and Cosine are cofunctions, which are complementary

$$\mathbf{sin}\boldsymbol{\theta} = \mathbf{cos}(\mathbf{90}^\circ - \boldsymbol{\theta})$$
$$\mathbf{cos}\boldsymbol{\theta} = \mathbf{sin}(\mathbf{90}^\circ - \boldsymbol{\theta})$$

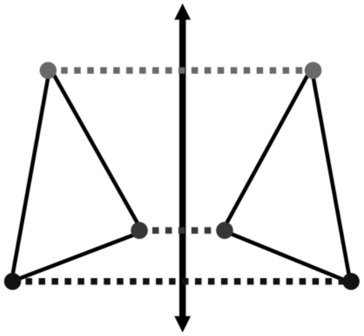
- If $\angle A$ and $\angle B$ are the acute angles of a right triangle, then $\mathbf{sin}\,\boldsymbol{A} = \mathbf{cos}\,\boldsymbol{B}$

It's calculator time!

TRANSFORMATIONAL GEOMETRY

Rigid Motion: a type of transformation that preserves distance, congruency, angle measure, size, and shape.

Reflection – FLIP



$$r_{x-axis}(x,y)=(x,-y)$$

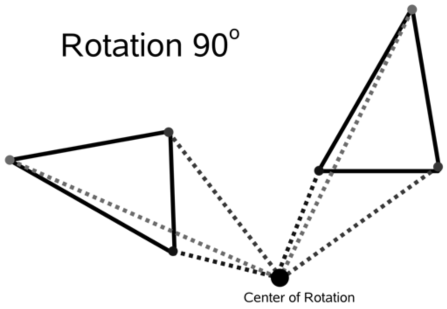
$$r_{y-axis}(x,y)=(-x,y)$$

$$r_{y=x}(x,y)=(y,x)$$

$$r_{y=-x}(x,y)=(-y,-x)$$

$$r_{(o,o)}(x,y)=(-x,-y)$$

Rotation – TURN

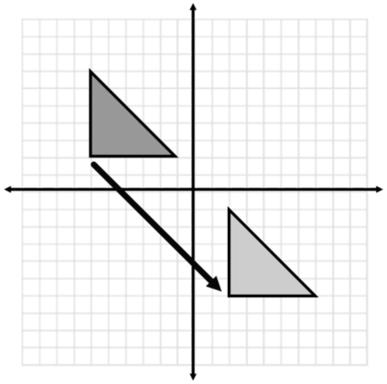


$$R_{90^\circ}(x,y)=(-y,x)$$

$$R_{180^\circ}(x,y)=(-x,-y)$$

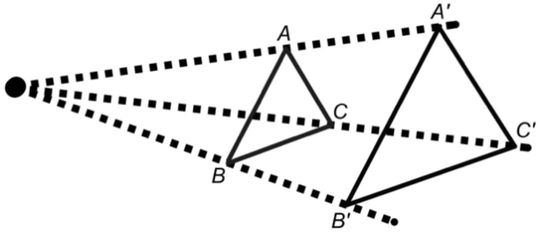
$$R_{270^\circ}(x,y)=(y,-x)$$

Translation – SHIFT/MOVE



$$T_{a,b}(x,y)=(x+a,y+b)$$

Dilation – ENLARGEMENT/REDUCTION



$$D_k(x,y)=(k\cdot x,k\cdot y)$$

- **Dilations create similar figures, where the corresponding sides are in proportion and the corresponding angles are congruent.**
- **Dilations are *not always* rigid motions, since they do *not always* preserve distance or congruency.**

Composition of Transformations

When you see "∘", work from right to left.

$$R_{90^\circ}\circ T_{3,-4}$$

Do this Second!

Do this First!

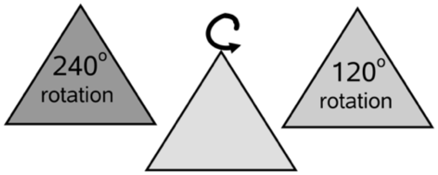
The example shows a translation to the right by three units and down by four units, followed by a rotation of 90 degrees.

Types of Composition Transformations

- A composition of 2 reflections over 2 parallel lines is equivalent to a **translation.**
- A composition of 2 reflections over 2 intersecting lines is equivalent to a **rotation.**

Rotational Symmetry Theorem

A regular polygon with n sides always has rotational symmetry, with rotations in increments equal to its central angle of $\frac{360^\circ}{n}$. Rotational symmetry is commonly referred as "mapping the figure onto itself".



CIRCLES

Circle Definition: A 2-dimensional shape made by drawing a curve that is always the same distance from the center.

Circle Equations

General/Standard Equation of a Circle:

$$x^2 + y^2 + Cx + Dy + E = 0$$

where C, D, and E are constants.

Center – Radius Equation of a Circle:

$$(x - h)^2 + (y - k)^2 = r^2$$

where *(h, k)* is the center and *r* is the radius.

Review of Factoring

The order of Factoring:

Greatest Common Factor **(GCF)**

↓

Difference of Two Perfect Squares **(DOTS)**

↓

Trinomial/"AM Method" **(TRI)**

GCF:

$$ab + ac \;=\; a(b + c)$$

DOTS:

$$x^2 - y^2 \;=\; (x + y)(x - y)$$

TRI:

$$x^2 - x + 6 \text{ » } (x + 2)(x - 3)$$

Completing the Square

The method of "completing the square" is used when factoring by the basic "Trinomial Method", or "AM" method cannot be applied to the problem. The completing the square method is commonly used in geometry ***to express a general circle equation in center-radius form.***

Example: Express the general equation $x^2 + 4x + y^2 - 6y - 12 = 0$ in center-radius form.

$$x^2 + 4x + y^2 - 6y - 12 = 0$$

$$x^2 + 4x + y^2 - 6y = 12$$

$$x^2 + 4x + \_ + y^2 - 6y + \_ = 12 + \_ + \_$$

$$x^2 + 4x + \mathbf{4} + y^2 - 6y + \mathbf{9} = 12 + \mathbf{4} + \mathbf{9}$$

$$(x + 2)(x + 2) + (y - 3)(y - 3) = 25$$

$$\boxed{(x + 2)^2 + (y - 3)^2 = 25}$$

Formula: $\left(\frac{b}{2}\right)^2$

Steps:

1) Determine if the squared terms have a coefficient of 1
2) If there is a constant/number on the left side of the equal sign, move that constant to the right side
3) Insert "boxes" or "blank spaces" after the linear terms to acquire a perfect-square trinomial
4) Take half of the linear term(s) and square the number. Insert this number on both the left and right sides
5) Factor using the "trinomial method"
6) Write your equation

Graphing Circles

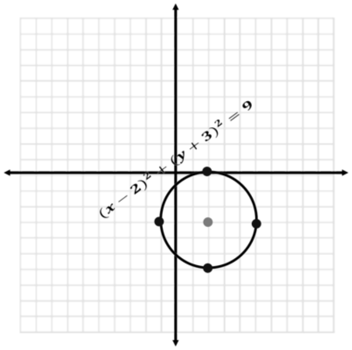
Steps:

1) Determine the center and the radius
2) Plot the center on the graph
3) Around the center, create four loci points that are equidistant from the center of the circle
4) Using a compass or steady freehand, connect all four points
5) Label when finished

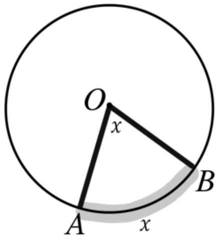
Example: Graph $(x - 2)^2 + (y + 3)^2 = 9$

The center is the point (2,-3)

The radius is 3

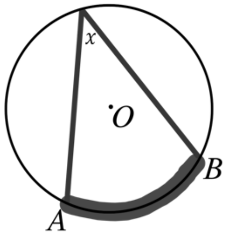


Angle Relationships in a Circle



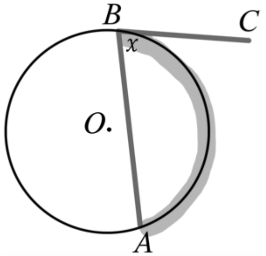
Central Angle:

$\measuredangle x = \widehat{AB}$



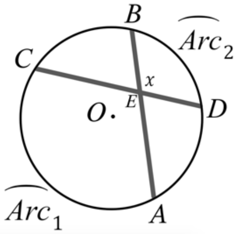
Inscribed Angle:

$\measuredangle x = \frac{1}{2}\widehat{AB}$



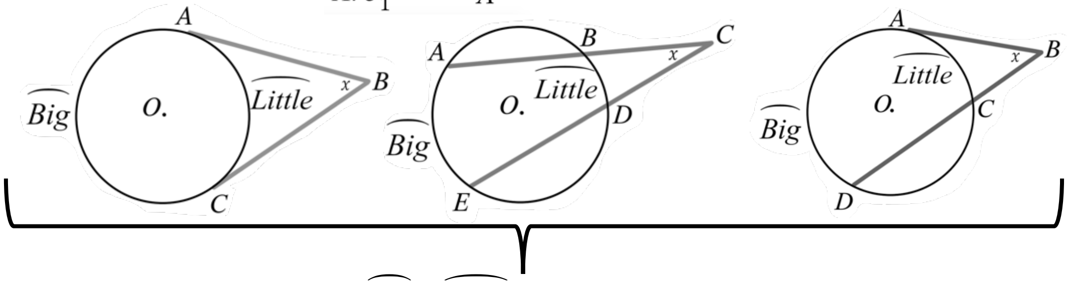
Tangent-Chord Angle:

$\measuredangle x = \frac{1}{2}\widehat{AB}$

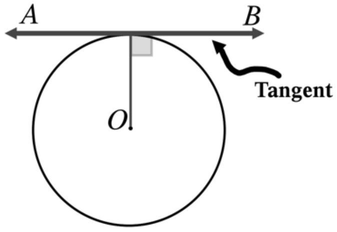


Two Chord Angles:

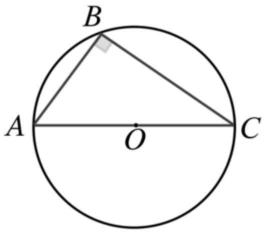
$\measuredangle x = \dfrac{\widehat{Arc_1} + \widehat{Arc_2}}{2}$



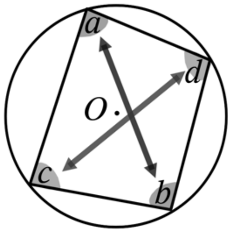
$\dfrac{\widehat{Big} - \widehat{Little}}{2} = \measuredangle x$



A tangent is perpendicular to its radius, forming a 90° angle



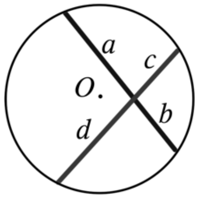
An angle that is inscribed in a semicircle equals 90°



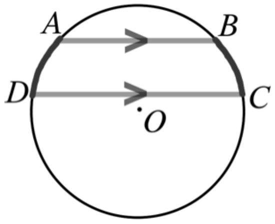
$a + b = 180°$
$c + d = 180°$

If a quadrilateral is inscribed in a circle, then its opposite angles = 180°

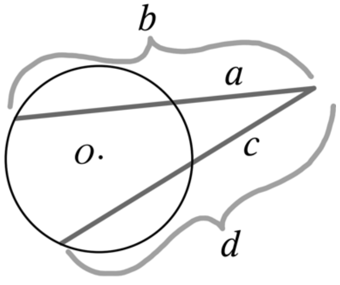
Segment Relationships in a Circle



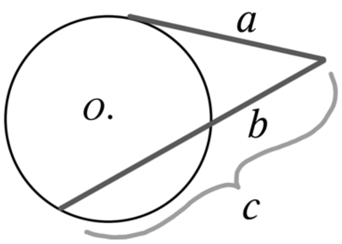
(Part)(Part)=(Part)(Part)
$(a)(b) = (c)(d)$



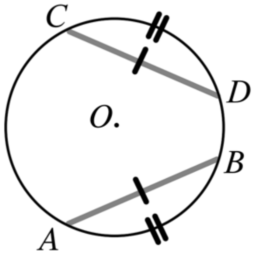
If $\overline{AB} \parallel \overline{DC}$, then $\widehat{AD} \cong \widehat{BC}$
Parallel chords intercept congruent arcs



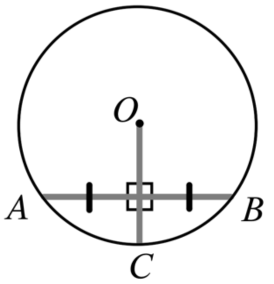
(W)(E) = (W)(E)
(Whole)(External)=(Whole)(External)
$(b)(a) = (d)(c)$



$(W)(E) = (T)^2$
$(Whole)(External)=(Tangent)^2$
$(c)(b) = (a)^2$



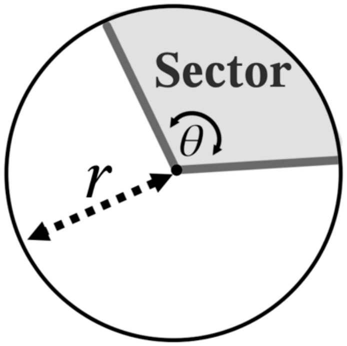
If $\overline{AB} \cong \overline{CD}$, then $\widehat{AB} \cong \widehat{CD}$



If a diameter/radius is perpendicular to a chord, then the diameter/radius bisects the chord and its arc.

Circles (Con't)

Area of a Sector



$$A = \frac{1}{2} r^2 \theta$$

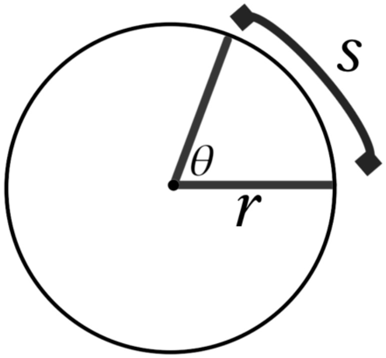
where A is the area of the sector, r is the radius, and θ is an angle in radians.

-or-

$$A = \frac{n}{360} \pi r^2$$

where A is the area of the sector, n is the amount of degrees in the central angle, and r is the radius

Sector Length

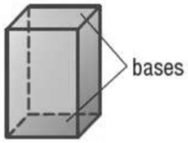


$$s = r \cdot \theta$$

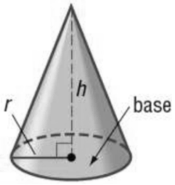
where s is the sector length, r is the radius, and θ is an angle in radians.

3-D FIGURES

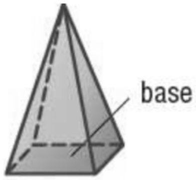
Prism



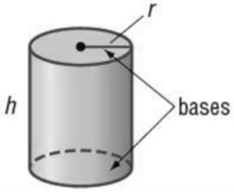
Cone



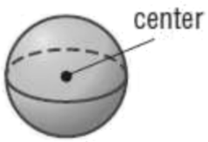
Pyramid



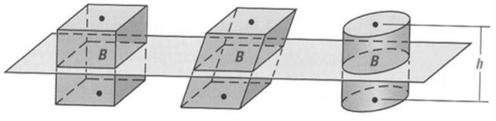
Cylinder



Sphere



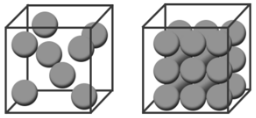
Cavalieri's Principle: If two solids have the same height and the same cross-sectional area at every level, then the solids have the same volume.



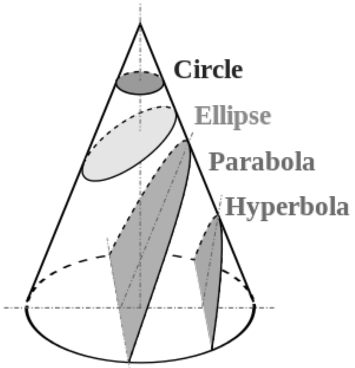
Density Formulas:

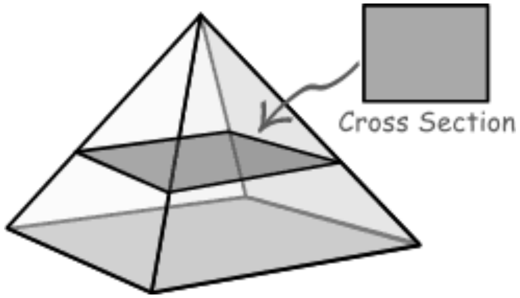
$$\boldsymbol{Mass} = (\boldsymbol{Density}) \cdot (\boldsymbol{Volume})$$

$$\boldsymbol{Density} = \frac{(\boldsymbol{Mass})}{(\boldsymbol{Volume})}$$



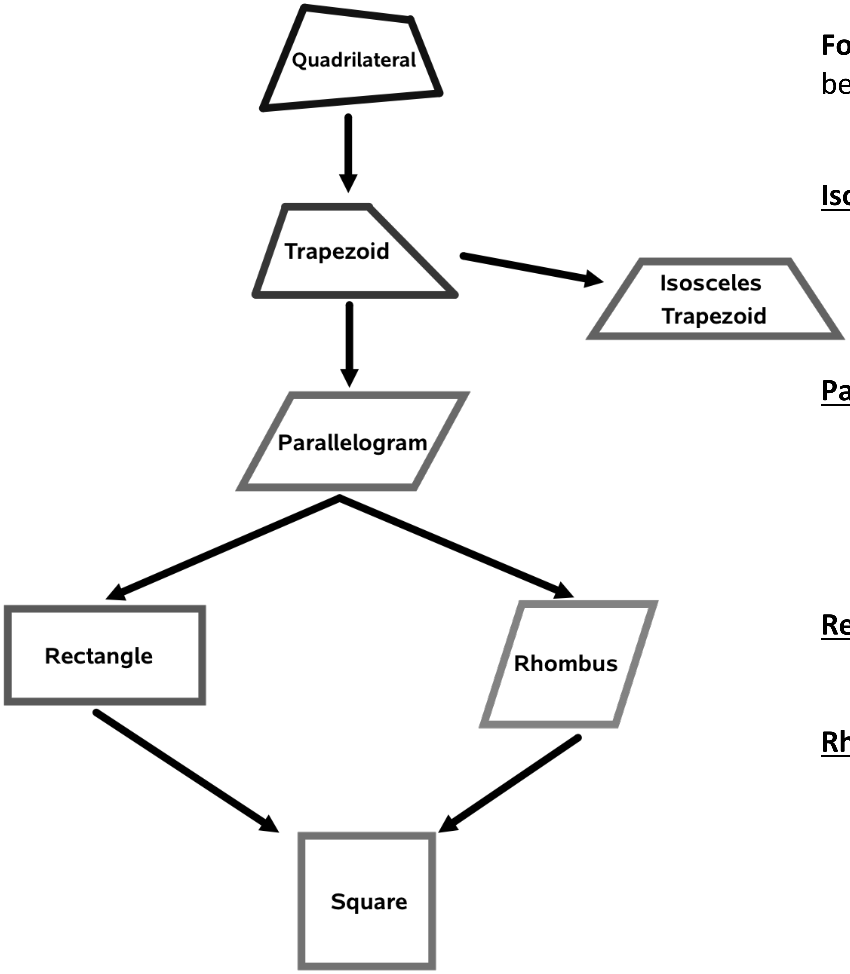
Cross Sections: a surface or shape that is or would be exposed by making a straight cut through something at one or multiple points.





QUADRILATERALS

The Quadrilateral Family Tree



The Quadrilateral Properties

Quadrilateral

- ✓ A quadrilateral is a four-sided polygon

Trapezoid

- ✓ at least one pair of parallel sides

Formula: The length of the **median** of a trapezoid can be calculated using the following formula:

$$Median = \frac{1}{2}(Base_1 + Base_2)$$

Isosceles Trapezoid

- ✓ each pair of base angles are congruent
- ✓ diagonals are congruent
- ✓ one pair of congruent sides (which are the called the *legs.* These are the non-parallel sides)

Parallelogram

- ✓ opposite sides are parallel
- ✓ opposite sides are congruent
- ✓ opposite angles are congruent
- ✓ consecutive angles are supplementary
- ✓ diagonals bisect each other

Rectangle

- ✓ all angles at its vertices are right angles
- ✓ diagonals are congruent

Rhombus

- ✓ all sides are congruent
- ✓ diagonals are perpendicular
- ✓ diagonals bisect opposite angles
- ✓ diagonals form four congruent right triangles
- ✓ diagonals form two pairs of two congruent isosceles triangles

Square

- ✓ diagonals form four congruent isosceles right triangles

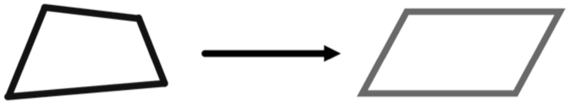
⚠ ⚠

Each figure *inherits the properties* of its parent

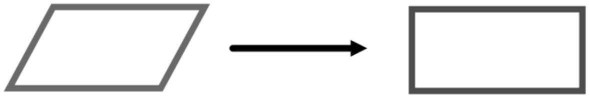
COORDINATE GEOMETRY PROOFS WITH POLYGONS

How to prove Quadrilaterals

- To prove that a *quadrilateral* is a *parallelogram*, it is sufficient to show any one of these properties:
  - ✓ Both pairs of opposite sides are parallel
  - ✓ Both pairs of opposite sides are congruent
  - ✓ Both pairs of opposite angles are congruent
  - ✓ One pair of opposite sides are both parallel and congruent
  - ✓ Diagonals bisect each other



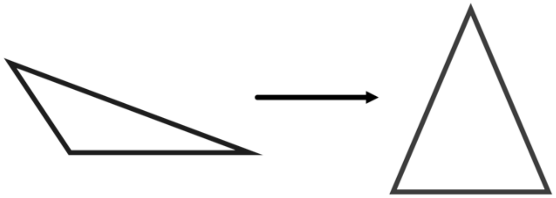
- To prove that a *parallelogram* is a *rectangle*, it is sufficient to show any one of these:
  - ✓ Any one of its angles is a right angle
  - ✓ One pair of consecutive angles are congruent
  - ✓ Diagonals are congruent



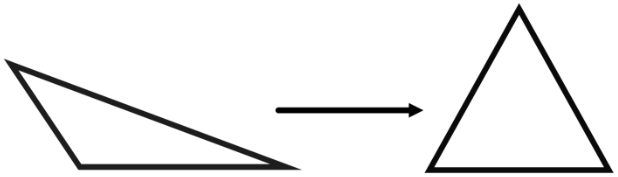
- To prove that a *parallelogram* is a *rhombus*, it is sufficient to show any one of these:
  - ✓ One pair of consecutive sides are congruent
  - ✓ Diagonals are perpendicular
  - ✓ Either diagonal is an angle bisector

How to prove Triangles

- To prove that a given triangle is an *isosceles* triangle, it is sufficient to show that two sides are congruent.



- To prove that a given triangle is an *equilateral* triangle, it is sufficient to show that all three sides are congruent.

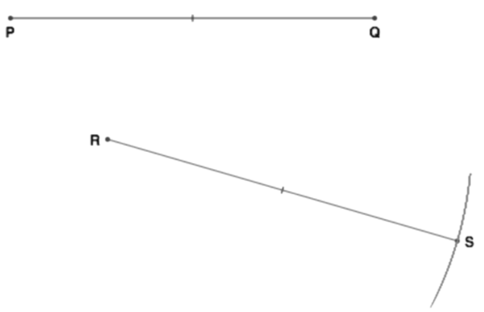


Remember – if there is a coordinate geometry proof on the regents, devise a plan, write it down, and use the coordinate geometry formulas shown in the "Coordinate Geometry" section of this packet to prove some properties!

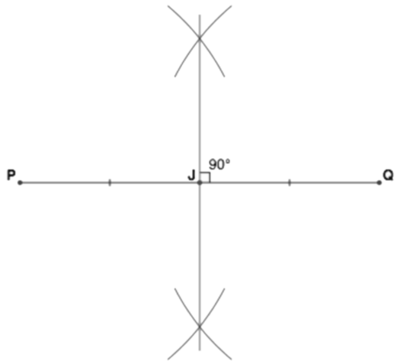
CONSTRUCTIONS

There will be either one or two constructions on the Geometry regents. It is important to understand basic constructions.

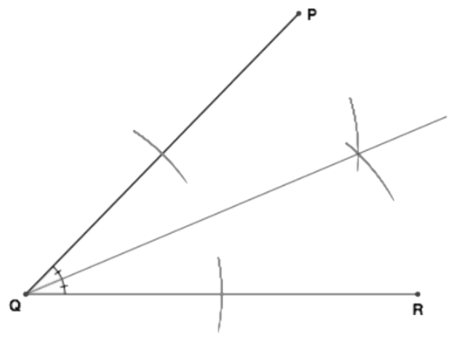
Copy a Line Segment



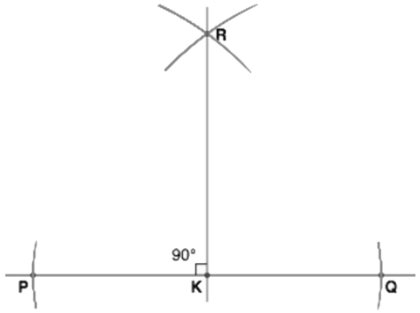
Perpendicular Bisector



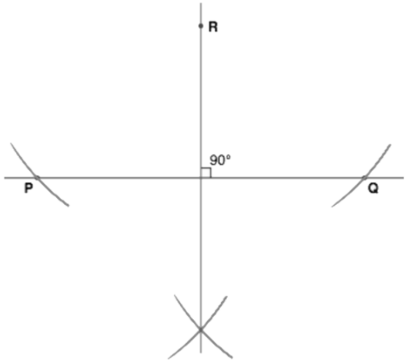
Angle Bisector



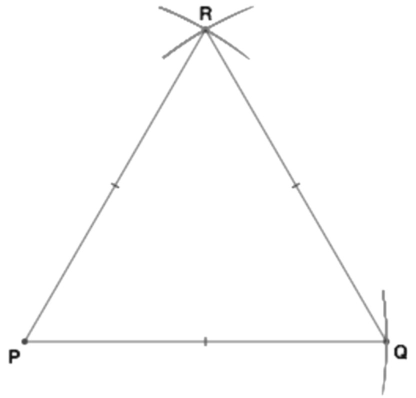
Perpendicular Line passing through a Point on the Given Line



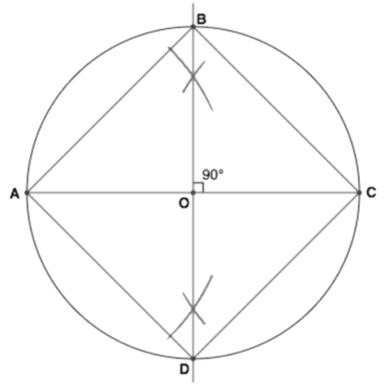
Perpendicular Line passing through a Point NOT on the Given Line



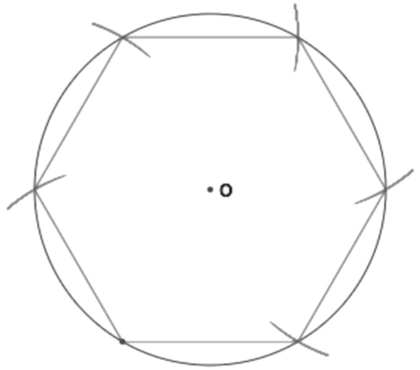
Equilateral Triangle



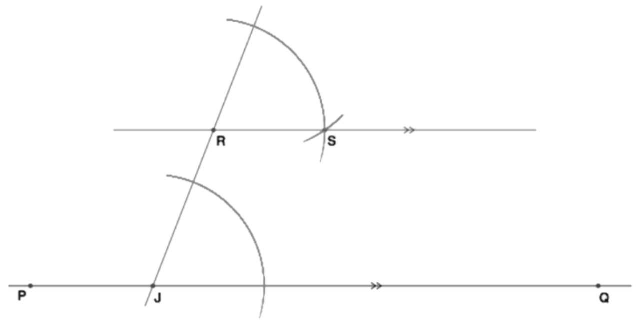
Square Inscribed in a Circle



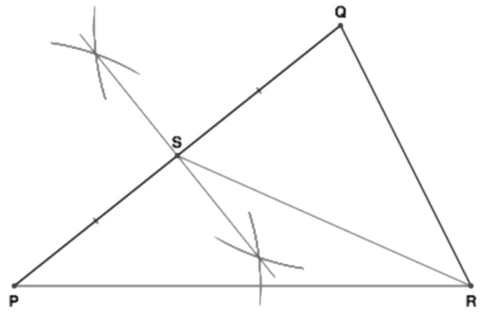
Hexagon Inscribed in a Circle



Parallel Lines



Median – A median is drawn to its midpoint



Centroid – The intersection of 3 medians